

Scalar Domain Wall as the Universe

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Abstract

By using the formulation of the reconstruction, we construct models which have an exact solution describing the domain wall. The shape of the domain wall can be flat, de Sitter space-time, or anti-de Sitter space-time. In the constructed domain wall solutions, there often appears ghost with negative kinetic energy. We give, however, an example of the de Sitter domain wall solution without ghost, which could be a toy model of the inflation. We also investigate the localization of the gravity as in the Randall-Sundrum model. It is shown that the four dimensional Newton law could be reproduced even in the de Sitter space-time domain wall solution.

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I. INTRODUCTION

There are many scenarios that our universe could be a brane in the higher dimensional space-time [1–5]. The inflationary brane world models were also considered by using the trace anomaly in [6–8]. Before the brane world scenario, there was a scenario that we live on the domain wall [9], and bent domain wall [10] as well as dynamical domain wall [11] were also investigated. After that there were many activities in the domain wall or thick brane universe scenario [12–17]. The domain wall has a thickness and the brane could be regarded as a limit that the thickness of the domain wall vanishes.

In this paper, we consider the domain wall by using the scalar field. We construct models where the shape of the domain wall is flat, de Sitter space-time, or anti-de Sitter space-time. In case that the shape of the domain wall is de Sitter space-time, if we regard the domain wall as our universe, the de Sitter space-time may express the inflation and/or the accelerating expansion of the current universe. By using the formulation of the reconstruction,¹ we give models which have an exact solution describing the domain wall. We also show that there appears massless graviton propagating on the four dimensional domain wall. The four dimensional massless graviton is the zero mode of the five dimensional graviton and the existence of the four dimensional massless graviton may tell that the four dimensional gravity, where the Newton potential behaves as $1/r$ for the distance r , could be reproduced on the domain wall. In the next section, we explain the formulation to obtain models which admit exact solutions describing the domain wall and we also give some examples. In the domain wall solutions, there appears ghost in general. The ghost has negative kinetic energy. We give, however, an example of de Sitter domain wall without ghost, which can be a toy model of the inflation. In Section III, we investigate the localization of the gravity and show the four dimensional Newton law could be reproduced. The last section is devoted to the summary and discussion. We use units of $k_B = c = \hbar = 1$ and denote the gravitational constant $8\pi G$ by $\kappa^2 \equiv 8\pi/M_{\text{Pl}}^2$ with the Planck mass of $M_{\text{Pl}} = G^{-1/2} = 1.2 \times 10^{19} \text{GeV}$.

¹ About the reconstruction for cosmology, see [18].

II. CONSTRUCTING THE ACTION GENERATING AN EXACT DOMAIN WALL SOLUTION

In this section, based on [19], we show how we can construct models which admit the exact solutions describing the domain wall. We use a procedure proposed in Ref. [20]. This formulation is a kind of reconstruction, that is, for an arbitrary warp factor, we specify the action of the Einstein gravity coupled with a scalar field which has the solution corresponding to the scale factor.

Our model action is as follows:

$$S = \int d^D x \sqrt{-g} \left(\frac{R}{2\kappa^2} - \frac{\omega(\varphi)}{2} \partial_\mu \varphi \partial^\mu \varphi - \mathcal{V}(\phi) \right). \quad (2.1)$$

We assume the $D = d + 1$ dimensional warped metric

$$ds^2 = dy^2 + L^2 e^{u(y)} \sum_{\mu, \nu=0}^{d-1} \hat{g}_{\mu\nu} dx^\mu dx^\nu, \quad (2.2)$$

with L being a dimensionless constant. We suppose that the scalar field only depends on y . In the metric (2.2), $\hat{g}_{\mu\nu}$ is the metric of the d -dimensional Einstein manifold defined by $\hat{R}_{\mu\nu} = [(d-1)/l^2] \hat{g}_{\mu\nu}$. The de Sitter space (the anti-de Sitter space) corresponds to $1/l^2 > 0$ ($1/l^2 < 0$), whereas the flat space to $1/l^2 = 0$. The (y, y) and (μ, ν) components of the Einstein equation are given by

$$-\frac{d(d-1)}{2l^2} e^{-u} + \frac{d(d-1)}{8} (u')^2 = \frac{1}{2} \omega(\varphi) (\varphi')^2 - \mathcal{V}(\varphi), \quad (2.3)$$

$$-\frac{(d-1)(d-2)}{2l^2} e^{-u} + \frac{d-1}{2} u'' + \frac{d(d-1)}{8} (u')^2 = -\frac{1}{2} \omega(\varphi) (\varphi')^2 - \mathcal{V}(\varphi). \quad (2.4)$$

Here the prime denotes the derivative with respect to y . Now φ may be chosen as $\varphi = y$. Moreover, we take $\kappa^2 = 1$. Then Eqs. (2.3) and (2.4) lead to

$$\omega(\varphi) = -\frac{d-1}{2} u'' - \frac{d-1}{l^2} e^{-u}, \quad (2.5)$$

$$\mathcal{V}(\varphi) = -\frac{d-1}{4} u'' - \frac{d(d-1)}{8} (u')^2 + \frac{(d-1)^2}{2l^2} e^{-u}. \quad (2.6)$$

Hence, the energy density ρ is now described as [19]

$$\rho = \frac{\omega(\varphi)}{2} (\varphi')^2 + \mathcal{V}(\varphi) = -\frac{d-1}{2} u'' - \frac{d(d-1)}{8} (u')^2 + \frac{(d-1)(d-2)}{2l^2} e^{-u}. \quad (2.7)$$

The pressure p_y for the y direction and the pressures p for other spatial directions are different with each other and given by

$$\begin{aligned} p_y &= \frac{\omega(\varphi)}{2} (\varphi')^2 - \mathcal{V}(\varphi) = \frac{d(d-1)}{8} (u')^2 - \frac{(d-1)(d-2)}{2l^2} e^{-u}, \\ p = \rho &= -\frac{d-1}{2} u'' - \frac{d(d-1)}{8} (u')^2 + \frac{(d-1)(d-2)}{2l^2} e^{-u}. \end{aligned} \quad (2.8)$$

Provided that the D dimensional space is asymptotically flat, we see $u \rightarrow 0$ in the limit of $|y| \rightarrow \infty$, the second term becomes dominant in (2.5), $\omega(\varphi) \sim -(d-1)/l^2$ if $1/l^2 \neq 0$. If $\omega(\varphi)$ is negative, which corresponds to the de Sitter space with $1/l^2 > 0$, the scalar field φ is ghost. For $1/l^2 = 0$, we obtain $\omega(\varphi) = -(d-1)u''/2$. Thus we impose the Z_2 symmetry of the metric, which is the invariance under the transformation $y \rightarrow -y$. In addition, we suppose the D dimensional space is asymptotically flat. In such a case, there must exist a region in which $\omega(\varphi)$ is negative and hence φ is ghost. We should also remark that the energy density often becomes negative. In any case, if we allow the ghost and negative energy density, for an arbitrary u , we find a model which permits u to be a solution of the Einstein equation. Furthermore, the problem of ghost can be avoided in case that the extra dimensions are compact.

As an example, we may examine

$$u = u_0 e^{-y^2/y_0^2}, \quad (2.9)$$

where u_0 and y_0 are constants. We explore the model

$$\begin{aligned} \omega(\varphi) &= -(d-1) \left(\frac{2\varphi^2}{y_0^4} - \frac{1}{y_0^2} \right) e^{-\varphi^2/y_0^2} - \frac{(d-1)}{l^2} e^{-u_0 e^{-\varphi^2/y_0^2}}, \\ \mathcal{V}(\varphi) &= -\frac{d-1}{2} \left(\frac{2\varphi^2}{y_0^4} - \frac{1}{y_0^2} \right) e^{-\varphi^2/y_0^2} + \frac{(d-1)^2}{l^2} e^{-u_0 e^{-\varphi^2/y_0^2}}. \end{aligned} \quad (2.10)$$

In this model, as a solution of the Einstein equation we acquire u in (2.9). Furthermore, we find the following distribution of the energy density [19]

$$\rho(y) = -\frac{d-1}{2} \left(\frac{2y^2}{y_0^4} - \frac{1}{y_0^2} \right) e^{-y^2/y_0^2} + \frac{(d-1)^2}{l^2} e^{-u_0 e^{-y^2/y_0^2}}. \quad (2.11)$$

As a result, the energy density $\rho(y)$ is localized at $y \sim 0$ and therefore a domain wall is made. We also mention that for $1/l^2 > 0$, the shape of the domain wall is a de Sitter space, and hence it could represent the accelerating universe.

A. (Anti-)de Sitter space-time

As a preparation to deal with the domain wall, here we consider the (anti-)de Sitter space-time and the flat space-time. These space-times do not correspond to brane or domain wall but we give explicit formula for later use.

We now study the de Sitter space-time solution. When $L^2 = l^2 > 0$ (L appears in (2.2)), if $u(y)$ is given by

$$u = 2 \ln \cosh \frac{y}{l}, \quad (2.12)$$

we find

$$\omega(\varphi) = 0, \quad \mathcal{V}(\varphi) = \frac{d(d-1)}{2l^2}. \quad (2.13)$$

We may also investigate the anti-de Sitter solution. When $L^2 = l^2 > 0$, if $u(y)$ is given by

$$u = 2 \ln \sinh \frac{y}{l}, \quad (2.14)$$

we acquire

$$\omega(\varphi) = 0, \quad \mathcal{V}(\varphi) = -\frac{d(d-1)}{2l^2}. \quad (2.15)$$

Here y is restricted to be $y \geq 0$. On the other hand, when $l^2 = -\tilde{l}^2 = -L^2 < 0$, if $u(y)$ is given by

$$u = 2 \ln \cosh \frac{y}{\tilde{l}}, \quad (2.16)$$

we have

$$\omega(\varphi) = 0, \quad \mathcal{V}(\varphi) = -\frac{d(d-1)}{2\tilde{l}^2}. \quad (2.17)$$

When $1/l^2 = 0$, if $u(y)$ is given by

$$u = \frac{2y}{L}, \quad (2.18)$$

where L is a constant, we obtain

$$\omega(\varphi) = 0, \quad \mathcal{V}(\varphi) = -\frac{d(d-1)}{2L^2}. \quad (2.19)$$

We now examine the flat space-time. When $l^2 > 0$, we find

$$u = 2 \ln \frac{y}{l}, \quad \omega(\varphi) = \mathcal{V}(\varphi) = 0. \quad (2.20)$$

Here $y \geq 0$. When $1/l^2 = 0$, we, of course, acquire

$$u = \omega(\varphi) = \mathcal{V}(\varphi) = 0. \quad (2.21)$$

B. Randall-Sundrum like model

In case of the second Randall-Sundrum model [2], we have

$$1/l^2 = 0, \quad u(y) = -\frac{2|y|}{L}. \quad (2.22)$$

This model shows the localization of the gravity on the four dimensional brane. Motivated by the model (2.22), we analyze the following model,

$$1/l^2 = 0, \quad u(y) = -\frac{2\sqrt{y^2 + y_0^2}}{L}. \quad (2.23)$$

Here y_0 is a constant and the Randall-Sundrum model (2.22) corresponds to this model in the limit of $y_0 \rightarrow 0$. Then we obtain

$$\omega(\varphi) = \frac{(d-1)y_0^2}{L(\varphi^2 + y_0^2)^{\frac{3}{2}}}, \quad \mathcal{V}(\varphi) = \frac{(d-1)y_0^2}{2L(\varphi^2 + y_0^2)^{\frac{3}{2}}} - \frac{d(d-1)\varphi^2}{2L^2(\varphi^2 + y_0^2)}. \quad (2.24)$$

Then the energy density is given by

$$\rho(y) = \frac{(d-1)y_0^2}{L(y^2 + y_0^2)^{\frac{3}{2}}} - \frac{d(d-1)y^2}{2L^2(y^2 + y_0^2)}. \quad (2.25)$$

In the limit of $y_0 \rightarrow 0$, the second term in (2.25) gives the negative cosmological constant corresponding to the anti-de Sitter space-time and the first term gives a δ -function:

$$\frac{(d-1)y_0^2}{L(y^2 + y_0^2)^{\frac{3}{2}}} \rightarrow \frac{2(d-1)}{L}\delta(y), \quad -\frac{d(d-1)y^2}{2L^2(y^2 + y_0^2)} \rightarrow -\Lambda \equiv -\frac{d(d-1)}{2L^2}. \quad (2.26)$$

On the other hand, p_y and p in (2.8) are given by

$$p_y = \frac{d(d-1)y^2}{2L^2(y^2 + y_0^2)} \rightarrow \Lambda = \frac{d(d-1)}{2L^2}, \quad p = \rho \rightarrow \frac{2(d-1)}{L}\delta(y) - \frac{d(d-1)}{2L^2}. \quad (2.27)$$

C. de Sitter domain wall and brane

We now explore the de Sitter domain wall and brane with $l^2 > 0$, which may correspond to the expanding universe. The four dimensional de Sitter brane solution in five dimensional anti-de Sitter space-time using the trace anomaly is given by [6–8].

Motivated by (2.14), we may study

$$u = 2 \sinh \left(\frac{\sqrt{y_0^2 + z_0^2} - \sqrt{y_0^2 + z^2}}{l'} \right), \quad l' \equiv \frac{lz_0}{\sqrt{y_0^2 + z_0^2}}. \quad (2.28)$$

Here y_0 and z_0 are positive constants and we assume $-z_0 \leq z \leq z_0$. In the limit of $y_0 \rightarrow 0$, we find

$$\sqrt{y_0^2 + z_0^2} - \sqrt{y_0^2 + z^2} \rightarrow z_0 - |z|, \quad (2.29)$$

which implies that the space-time given by (2.28) in this limit can be obtained as follows: First cut two anti-de Sitter spaces given by (2.14) at $y = z_0$. Second glue two space-times with the region $0 \leq y \leq z_0$ at $y = z_0$. Then in the limit (2.29), $z = 0$ corresponds to $y = z_0$ and $z = \pm z_0$ to $y = 0$. Then we find

$$\begin{aligned} \omega(\varphi) &= (d-1) \left(\frac{\frac{\varphi^2}{l'^2(\varphi^2 + y_0^2)}}{\sinh^2 \left(\frac{\sqrt{y_0^2 + z_0^2} - \sqrt{y_0^2 + \varphi^2}}{l'} \right)} - \frac{y_0^2}{l'(\varphi^2 + y_0^2)^{\frac{3}{2}}} \frac{\cosh \left(\frac{\sqrt{y_0^2 + z_0^2} - \sqrt{y_0^2 + \varphi^2}}{l'} \right)}{\sinh \left(\frac{\sqrt{y_0^2 + z_0^2} - \sqrt{y_0^2 + \varphi^2}}{l'} \right)} \right) \\ &\quad - \frac{d-1}{l^2 \sinh^2 \left(\frac{\sqrt{y_0^2 + z_0^2} - \sqrt{y_0^2 + \varphi^2}}{l'} \right)}, \\ \mathcal{V}(\varphi) &= \frac{(d-1)}{2} \left(\frac{\frac{\varphi^2}{l'^2(\varphi^2 + y_0^2)}}{\sinh^2 \left(\frac{\sqrt{y_0^2 + z_0^2} - \sqrt{y_0^2 + \varphi^2}}{l'} \right)} - \frac{y_0^2}{l'(\varphi^2 + y_0^2)^{\frac{3}{2}}} \frac{\cosh \left(\frac{\sqrt{y_0^2 + z_0^2} - \sqrt{y_0^2 + \varphi^2}}{l'} \right)}{\sinh \left(\frac{\sqrt{y_0^2 + z_0^2} - \sqrt{y_0^2 + \varphi^2}}{l'} \right)} \right) \\ &\quad - \frac{d(d-1)}{2} \frac{\varphi^2}{l'^2(\varphi^2 + y_0^2)} \frac{\cosh^2 \left(\frac{\sqrt{y_0^2 + z_0^2} - \sqrt{y_0^2 + \varphi^2}}{l'} \right)}{\sinh^2 \left(\frac{\sqrt{y_0^2 + z_0^2} - \sqrt{y_0^2 + \varphi^2}}{l'} \right)} \\ &\quad + \frac{d-1}{2l^2 \sinh^2 \left(\frac{\sqrt{y_0^2 + z_0^2} - \sqrt{y_0^2 + \varphi^2}}{l'} \right)}. \end{aligned} \quad (2.30)$$

Here we have identified z as φ . We should note that $\omega(\varphi)$ is not always positive and hence the scalar field becomes ghost.

We now show that we can construct a model without ghost. For simplicity, we only examine the case $D = d + 1 = 5$. Let us explore the following model:

$$e^{u(y)} = \frac{y^2}{l^2} \left(1 + \frac{y^4}{y_0^4} \right)^{-1}. \quad (2.31)$$

When $y \rightarrow 0$, $e^{u(y)}$ behaves as $e^{u(y)} \sim y^2/l^2$. Therefore we can regard y as a radial coordinate and $y = 0$ corresponds to the center of the manifold whose topology of the spatial part is a four dimensional sphere. Since $d(e^{u(y)})|_{y=y_0} = 0$, we can cut the manifold at $y = y_0$ and

glue two copies of the manifold cut at $y = y_0$. For the model (2.31), we find

$$\begin{aligned}\omega(\varphi) &= 3 \left(1 + \frac{\varphi^4}{y_0^4}\right)^{-2} \frac{\varphi^2}{y_0^4} \left(5 + \frac{\varphi^4}{y_0^4}\right) \left(1 - \frac{\varphi^4}{y_0^4}\right), \\ \mathcal{V}(\varphi) &= \left(1 + \frac{\varphi^4}{y_0^4}\right)^{-2} \left(\frac{75\varphi^2}{2y_0^4} + \frac{6\varphi^6}{y_0^8} + \frac{9\varphi^{10}}{2y_0^{12}}\right).\end{aligned}\tag{2.32}$$

When $y^2 = \varphi^2 \leq y_0^2$, both $\omega(\varphi)$ and $\mathcal{V}(\varphi)$ are positive and therefore there does not appear ghost. In order to avoid the ghost, we may restrict the value of φ to be $\varphi^2 \leq y_0^2$. The energy density is also given by

$$\rho(y) = \left(1 + \frac{y^4}{y_0^4}\right)^{-2} \left(\frac{45y^2}{y_0^4} + \frac{3y^{10}}{y_0^{12}}\right),\tag{2.33}$$

which vanishes at $y = 0$ and localizes at $y = y_0$. Then we have constructed a model of the de Sitter domain wall without ghost.

We may remark that, when we glue the two copies of the region $y^2 \leq y_0^2$, the value of d^3u/dy^3 becomes discontinuous at $y = y_0$ although the values of u , u' , and u'' are continuous. This means that the derivative of the curvatures with respect to y and therefore the derivative of the energy density in (2.33) become discontinuous at $y = y_0$. We should note, however, that this discontinuities never conflicts with the Einstein equation and field equation since these equations do not contain the derivative higher than two.

III. LOCALIZATION OF GRAVITY

As executed in [2], we now investigate the localization of the gravity. Here we restrict the consideration to the case of $d = 4$ for simplicity. For this purpose, we examine the following perturbation

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}.\tag{3.1}$$

Here $g_{\mu\nu}^{(0)}$ is the metric given in the previous sections by solving the Einstein equations, etc. We express the quantities given by $g_{\mu\nu}^{(0)}$ by using the suffix (0) and we define the lowering and raising of the vector index by using $g_{\mu\nu}^{(0)}$ and $g^{(0)\mu\nu}$ like $h^\mu{}_\nu = g^{(0)\mu\rho} h_{\rho\nu}$.

We have

$$\begin{aligned}
\sqrt{-g} &= \sqrt{-g^{(0)}} \left(1 + \frac{1}{2} h_\mu^\mu + \frac{1}{8} (h_\mu^\mu)^2 - \frac{1}{4} h_{\mu\nu} h^{\mu\nu} + \mathcal{O}(h^3) \right), \\
R &= R^{(0)} - R^{(0)\mu\nu} h_{\mu\nu} + \nabla^{(0)\mu} \nabla^{(0)\nu} h_{\mu\nu} - \nabla^{(0)2} h_\mu^\mu \\
&\quad + \frac{3}{4} (\nabla^{(0)\sigma} h_\rho^\rho) \nabla^{(0)\nu} h_{\sigma\nu} + \frac{3}{4} h^{\mu\nu} \nabla_\mu^{(0)} \nabla_\nu^{(0)} h_\rho^\rho - \frac{1}{2} \nabla^{(0)\mu} h_{\mu\lambda} \nabla_\nu^{(0)} h^{\nu\lambda} \\
&\quad + \frac{1}{4} h^{\mu\nu} \nabla^{(0)2} h_{\mu\nu} - \frac{1}{4} (\nabla^{(0)\sigma} h_\rho^\rho) (\nabla_\sigma^{(0)} h_\mu^\mu) + \frac{1}{2} R_{\mu\nu}^{(0)} h^{\mu\rho} h^\nu_\rho + \frac{1}{2} R^{(0)\rho\mu\sigma\nu} h_{\rho\sigma} h_{\mu\nu} \\
&\quad + \nabla^{(0)\mu} \left(-\frac{1}{4} h_{\kappa\nu} \nabla_\mu h^{\nu\kappa} - \frac{1}{2} h_{\mu\kappa} \nabla_\rho^{(0)} h^{\rho\kappa} + \frac{1}{4} h_{\mu\kappa} \nabla^{(0)\kappa} h_\rho^\rho \right) \\
&\quad + \frac{1}{2} \nabla^{(0)2} (h_{\mu\nu} h^{\mu\nu}) + \mathcal{O}(h^3). \tag{3.2}
\end{aligned}$$

Since we are interested in the localization of the graviton, which is massless and a spin two particle, we now assume,

$$h_{0\mu} = h_{y\mu} = \nabla^{(0)i} h_{ij} = h_i^j = 0. \tag{3.3}$$

In the following, we only study $D = d + 1 = 5$ case for simplicity. Thus the action is reduced as follows:

$$\begin{aligned}
&\int d^5x \sqrt{-g} \left[\frac{R}{2\kappa^2} - \frac{1}{2} \omega(\varphi) \partial_\mu \varphi \partial^\mu \varphi - \mathcal{V}(\varphi) \right] \\
&\rightarrow \int d^Dx \sqrt{-g^{(0)}} \left[\frac{1}{2\kappa^2} \left\{ R^{(0)} - \frac{1}{4} \nabla^{(0)\rho} h^{ij} \nabla_\rho^{(0)} h_{ij} + \frac{1}{2} R^{(0)ij} h_{ik} h_j^k + \frac{1}{2} R^{(0)ikjl} h_{ij} h_{kl} \right. \right. \\
&\quad \left. \left. - \frac{1}{4} R^{(0)} h_{ij} h^{ij} \right\} - \frac{1}{4} h_{ij} h^{ij} \left(-\frac{1}{2} \omega(\varphi) \partial_\mu \varphi \partial^\mu \varphi - \mathcal{V}(\varphi) \right) + \mathcal{O}(h^3) \right]. \tag{3.4}
\end{aligned}$$

Then by the variation of h_{ij} , we obtain

$$\begin{aligned}
0 &= \frac{1}{2\kappa^2} \left\{ \frac{1}{2} \nabla^\rho \nabla_\rho h_{ij} + \frac{1}{2} \left(R_{ik}^{(0)} h_j^k + R_{jk}^{(0)} h_i^k \right) \right. \\
&\quad \left. + R_{ikjl}^{(0)} h^{kl} - \frac{1}{2} R^{(0)} h_{ij} \right\} - \frac{1}{2\kappa^2} h_{ij} \left(-\frac{1}{2} \omega(\varphi) \partial_\mu \varphi \partial^\mu \varphi - \mathcal{V}(\varphi) \right). \tag{3.5}
\end{aligned}$$

First we explore the case that the domain wall is flat, i.e., $\hat{g}_{\mu\nu} = \eta_{\mu\nu}$ in (2.2). Since

$$\nabla^\rho \nabla_\rho h_{ij} = (\partial_y^2 + e^{-u} \square) h_{ij} - \left(u'' + \frac{3}{2} u'^2 \right) h_{ij}, \tag{3.6}$$

we find

$$0 = (\partial_y^2 + e^{-u} \square) h_{ij} + \left(-u'' - u'^2 \right) h_{ij}. \tag{3.7}$$

Here we have used (2.5) and (2.6) with $d = 4$. We should note that there is always a zero-mode solution, where $\square h_{ij} = 0$ in the flat (domain wall) space-time. The explicit form of

the zero-mode is given by

$$h_{ij} \propto e^u, \quad (3.8)$$

which could surely be normalizable if u sufficiently rapidly goes to minus infinity $u \rightarrow +\infty$ when $|y| \rightarrow \infty$.

Next we investigate the case that the domain wall is de Sitter or anti-de Sitter space-time, namely, $\hat{R}_{\mu\nu} = \frac{3}{l^2}\hat{g}_{\mu\nu}$. Then instead of (3.7), we obtain

$$0 = (\partial_y^2 + e^{-u}\square) h_{ij} + \left(-u'' - u'^2 - \frac{2e^{-u}}{l^2}\right) h_{ij}. \quad (3.9)$$

Here we have used (2.5) and (2.6) with $d = 4$. In the (anti)-de Sitter space-time, the zero-mode solution, which corresponds to the massless graviton, is given by

$$\square h_{ij} = \frac{2}{l^2} h_{ij}. \quad (3.10)$$

Therefore the zero-mode is again given by (3.8). Accordingly the zero-mode is always proportional to the warp factor e^u . As a result, even in the (anti-)de Sitter domain wall, there occurs the localization of graviton.

IV. SUMMARY AND DISCUSSIONS

By using the formulation of the reconstruction, we have found the models which have an exact solution describing the domain wall. The shape of the domain wall can be flat, de Sitter space-time, or anti-de Sitter space-time. In the domain wall solutions, there often appears ghost with negative kinetic energy. We have constructed, however, an example of the de Sitter domain wall solution without ghost, which can be a toy model of the inflation. We have also investigated the localization of the gravity and it has been shown that the four dimensional Newton law could be reproduced.

We have not, however, studied if the domain wall solution is stable or unstable. About the previous work on the stability of the domain wall, see [16].

For the check of the (in)stability, we need to consider the time-dependent perturbation from the solution. The existence of the massless graviton, which is obtained from the fluctuation of the metric, may inform that the model could be stable under the perturbation of the metric. In order to show the stability, however, we of course need to include the perturbation of the scalar field φ , which could be a future work.

Although we have constructed a model which has an exact de Sitter domain wall solution without ghost, the model is merely a toy model. Therefore the construction of more realistic model could also be one of future works.

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